University of Munich Department of Economics M.Sc. in Economics - Test of Admission (Example Problem Set)

A Microeconomics

1. Optimization

A monopolist produces the good x with costs of c for each unit of x. The inverse demand function is given by p(x) = a - bx (with a, b > 0).

- a) The monopolist has to pay a tax t (with 0 < t < 1) on his profits. What is the monopolist's optimization problem? Calculate the profit maximizing price and quantity from the first-order conditions.
- b) Now suppose that the monopolist has to pay the tax t not on his profits but on his revenues. What is now the monopolist's optimization problem? Again, calculate the profit maximizing price and quantity from the firstorder conditions.
- c) Suppose the relevant tax regime is the one from question b). How does the monopolist's profit change with a marginal increase in the tax rate t? Calculate the quantitative effect (i.e. the derivative) of an increase in t on profits and indicate the qualitative effect (i.e. whether profit increases or decreases).
- d) Which theorem can be used in question c) that simplifies the calculation and allows to disregard the indirect effects of a tax increase on the value function (i.e. the optimal profit function)?
- e) Now suppose that, due to a drop in the supply of input factors, the monopolist can now produce a maximum quantity of $\bar{x} = \frac{a(1-t)-c}{2b}$. Assume the tax regime from question b). Prove that the restriction \bar{x} is binding for $a > \frac{c}{1-t}$.

2. Adverse Selection

Consider the market for coffee machines. There are 200 risk-neutral buyers and 160 risk-neutral sellers. Each buyer wants to buy at most one coffee machine; each seller owns exactly one coffee machine.

There are two types of coffee machines: high quality and low quality machines. High quality machines have a failure probability of 0.2, whereas the low quality machines have a failure probability of 0.75.

The utility that a buyer derives from a coffee machine without failure amounts to 400 (measured in monetary terms). If the coffee machine has a failure the utility of the buyer decreases – by the amount of the repair costs – to 200.

Assume that 25% of the coffee machines are of high quality. Each seller has a reservation price of 300 for a high quality machine and a reservation price of 240 for a low quality machine.

a) Derive a buyer's maximum willingness to pay for a high quality and a low quality coffee machine.

Suppose that sellers know the quality of their machines, whereas the buyers **can-not** distinguish between high and low quality machines (asymmetric information).

- b) Derive aggregate supply and aggregate demand as a function of the market price.
- c) Characterize the market outcome. Comment briefly on its efficiency.
- d) How large may the failure probability of the low quality machines maximally be, so that there is just **no** partial market breakdown?

B Macroeconomics

Please highlight the correct answer. For each question there exists one correct answer.

 An open economy's goods market is described by the following equations: Consumption function:

$$C = a + b(Y - T)$$
 with $0 < b < 1$ and $a > 0$

Tax revenue:

$$T = tY$$
 with $0 \le t < 1$

Investment and government spending are exogenous: $I = I_0$, $G = G_0$. Net export function:

$$X = g - mY$$
 with $0 \le m < 1$ and $g > 0$

Let m = 0 and b = 0.9. An increase in the tax rate t from 0 to 0.3

- (a) should increase the investment multiplier $\frac{\Delta Y}{\Delta I_0}$ by around 73%.
- (b) should reduce the investment multiplier by around 73%.
- (c) should increase the government spending multiplier $\frac{\Delta Y}{\Delta G_0}$ by around 33%.
- (d) should reduce the government spending multiplier by around 33%.
- (e) should reduce both the investment and government spending multipliers by around 33%.

2. Suppose an open economy is described in the short run by the following standard IS-LM model:

IS-curve (goods market equilibrium condition):

$$R = \frac{a + e + g + G}{n + d} - \frac{1 - b(1 - t) + m}{n + d}Y \quad with \quad 0 < b < 1 \quad and \quad 0 \le t < 1$$

LM-curve (money market equilibrium condition):

$$R = -\frac{M}{hP} + \frac{k}{h}Y$$

M is the money supply, G is government spending, P is the price level. The remaining parameters a, e, g, n, d, m, k, h are all positive.

An expansionary monetary policy,

- (a) decreases the equilibrium output Y.
- (b) reduces the investment demand.
- (c) is more effective (in terms of output changes), if there is a higher tax rate t.
- (d) is more effective (in terms of output changes), if consumers have a lower propensity to consume (low b).
- (e) is more effective (in terms of output changes), if investment demand is more sensitive to interest rate changes (high d).
- 3. Let Y = 7700 7P represents an aggregate demand curve. If potential GDP were 7000, the price level that would produce aggregate demand just supporting that potential would be
 - (a) 110
 - (b) 100
 - (c) 90
 - (d) 105
 - (e) 75

4. Suppose an economy is described by the aggregate demand curve:

$$Y = 2000 + 1.25 * G + 2.5 * \frac{M}{P}.$$

If Y^* (the potential output) is equal to 6000, G = 1200, and P = 1, what must M equal to generate GDP, which is 3% lower than the potential.

- (a) 1072
- (b) 1000
- (c) 982
- (d) 928
- (e) impossible to calculate
- 5. Suppose that the share of GDP paid to capital was always equal to 25% and the remaining 75% was going to labor. That is,

$$Y_t = A_t K_t^{0.25} L_t^{0.75},$$

where A_t is total factor productivity. If, over the course of 20 years, the capital stock had been growing at 2% per year, the labor force had been growing at 3% per year, and GDP had been climbing at a 3% per year, then total factor productivity must have been

- (a) growing at 7% per year.
- (b) growing at 5% per year.
- (c) growing at 0.25% per year.
- (d) falling at 5% per year.
- (e) falling at 0.75% per year.

C Empirical Economics

1. Hypothesis testing in the linear regression model

You analyze a data set with 222 observations on the salaries of university professors and a few explanatory variables. Specifically, the data set contains these variables:

Annual salary in \$1000
Work experience (after PhD graduation) in years
Work experience (after PhD graduation) in years, squared
Dummy variable:
= 1 if the professor is affiliated with a private university
=0 if the professor is affiliated with a public university

Observations are indexed by i = 1, ..., N, with N = 222.

Consider the following regression model:

$$SALARY_i = \beta_0 + \beta_1 YEARS_i + \beta_2 PRIVATE_i + u_i$$
(1)

Suppose that the standard assumptions of the linear regression model with multiple explanatory variables are satisfied. In particular, assume that the error term, u_i , satisfies the conditional independence assumption $E(u_i | \text{YEARS}_i, \text{PRIVATE}_i) = 0$.

- (a) How can you test the hypothesis that two additional years of work experience have the same effect on the annual salary as being affiliated with a private university? Write down the null hypothesis and the name of the statistical test you would use.
- (b) You are interested in the difference in annual salaries between private and public universities. How can you use the results from estimating regression model (1) to compute the difference in the average salaries in private and public universities? Briefly explain your answer.

Next, consider a different regression model:

$$SALARY_i = \beta_0 + \beta_1 YEARS_i + \beta_2 YEARS2_i + u_i$$
(2)

Suppose that the standard assumptions of the linear regression model with multiple explanatory variables are satisfied in this model. Assume now that there is no salary difference between private and public universities. Thus, you may assume that the error term, u_i , satisfies the conditional independence assumption $E(u_i|\text{YEARS}_i, \text{YEARS}_i) = 0.$

You run the OLS regression with standard software and obtain the following output:

OLS using observations 1–222 Dependent variable: SALARY Heteroskedasticity-robust standard errors

	Coefficient	Std. error	<i>t</i> -statistic	<i>p</i> -value
const	41,9686	2,11574	19,8364	0,0000
YEARS	3,03331	0,381938	7,9419	0,0000
YEARS2	-0,0401094	0,0105401	-3,8054	0,0002

(c) Compute the average salary of a university professor with 10 years of work experience. (You should be able to perform this calculation without using a pocket calculator! You may round to full dollars in intermediate steps, provided you write down these calculations.)

2. True/false statements

Please indicate whether each statement is true or false by checking the appropriate box.

2.1 Linear regression model with one explanatory variable

The following statements refer to the linear regression model with one explanatory variable, i.e.

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

Observations are indexed by i = 1, ..., N, where N is the sample size.

We denote the parameter values estimated by OLS by $\hat{\beta}_0$ and $\hat{\beta}_1$.

The residual of observation *i* is defined as $\widehat{u}_i = y_i - \widehat{\beta}_0 - \widehat{\beta}_1 X_i$.

Please assume that the standard assumptions of the linear regression model hold, in particular, the error term satisfies the conditional independence assumption $E(u_i|X_i) = 0$. However, do *not* assume that the error term is normally distributed.

- (a) The OLS estimator $\hat{\beta}_1$ is unbiased even when the error term is not normally distributed.
 - [] true [] false
- (b) The OLS estimator $\hat{\beta}_1$ is consistent in small samples.
 - [] true [] false
- (c) The sum of the residuals, ∑^N_{i=1} û_i, is always equal to 0.
 [] true [] false
- (d) The larger the variance of X, the more precisely can β₁ be estimated.
 [] true
 [] false
- (e) The error term, u, includes all variables that influence X.
 - [] true [] false

2.2 Linear regression model with multiple explanatory variables

The following statements refer to the linear regression model with multiple explanatory variables, i.e.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \dots + \beta_k X_{ki} + u_i.$$

Observations are indexed by i = 1, ..., N, where N is the sample size.

We denote the parameter values estimated by OLS by $\widehat{\beta}_0$ and $\widehat{\beta}_1, \ldots, \widehat{\beta}_k$.

- (a) The OLS estimator $\hat{\beta}_1$ is consistent if only if X_1 is uncorrelated with all the other explanatory variables (X_2, \ldots, X_k) .
 - [] true [] false
- (b) It is not possible to calculate the OLS estimators $\hat{\beta}_1, \ldots, \hat{\beta}_k$ if one of the explanatory variables is a linear function of the remaining explanatory variables.
 - [] true [] false
- (c) In case of heteroskedasticity, it is not possible to calculate *t*-statistics.
 - [] true [] false
- (d) The OLS estimator $\hat{\beta}_1$ is inconsistent if the explanatory variable X_1 is correlated with the error term u.
 - [] true [] false
- (e) The difference between the coefficient of determination, R^2 , and the adjusted coefficient of determination (which is often denoted by \bar{R}^2) becomes smaller as the sample size increases.
 - [] true [] false

D Mathematical Methods

1. Differentiation and Integration

Differentiate the following functions with respect to x:

- a) $f(x) = \frac{10x}{x^2+3}$
- b) $f(x) = 5^x$
- c) $f(x) = e^{\frac{6}{x^2 + lnx}}$

d)
$$F(x) = \int_x^1 \sqrt{t} dt$$

Calculate the following integrals:

e) $\int \frac{9}{x} dx$ f) $\int_2^9 e^x dx$